Tips for Solving ***Conditional*** Trigonometric Equations

Trigonometric equations fall into one of two categories: *identities* where the equation is true for all values of the variable (we "establish" trigonometric identities), and *conditional equations* where the equation is true for only certain values of the variable (we "solve" conditional trigonometric equations).

Algebraic Techniques Used to Simplify Conditional Trigonometric Equations

Our goal is to use standard algebraic techniques to simplify an equation until it reaches one of two forms:

1. a linear equation consisting of a *single* trigonometric function equal to a constant, or
2. a *factorable* equation consisting of one or more trigonometric functions set equal to zero.

Algebraic techniques that can be used to simplify a trigonometric equation into one of these two forms include:

- Add/subtract the same constant or variable term from both sides of the equation.
- Multiply/divide both sides of the equation by the same NON–ZERO constant or variable term.
    - *Note*: When multiplying or dividing by a variable term you will need to put on a restriction that the term is not equal to zero.
    - *Note*: Also, if the original equation contains $\tan\theta$, $\csc\theta$, $\sec\theta$, or $\cot\theta$ then you will also need to put on a restriction that specifies that the denominator is not equal to zero.
- Take the square root of both sides of an equation and be sure to include the "±" symbol.

Working with Factorable Trigonometric Equations

If the trigonometric equation appears to be something other than a linear equation consisting of a single trigonometric function, then it should be factorable once we get it in the right form. Try to get the trigonometric equation to look like a polynomial in a *single variable*, such as all $\cos\theta$, then check to see if you have a …

- Quadratic binomial where you just need to factor out the GCF.
- Quadratic binomial that factors as a Difference of Perfect Squares.
- Quadratic trinomial that factors into the product of two binomials using standard factoring techniques.

In order to get your trigonometric equation consisting of a single variable term you might need to use a formula and simplify. If the trigonometric equation has …

- Two or more trigonometric functions and one of them is quadratic, try using one of the Pythagorean Identities, then simplify to get a factorable equation.
- A sum or difference of terms, and involves $\sin\theta$ and/or $\cos\theta$, try a Sum–to–Product formula to get a factorable product.
- Two or more different functions involving two different angles such as …
    - θ and 2θ, keep the θ then use a Double Angle Formula on the 2θ.
    - 2θ and 4θ, keep the 2θ then use a Double Angle Formula on $4\theta = 2[2\theta]$.
- An angle of $-\theta$, use your Even–Odd Identities to replace it with a trigonometric function in θ.
- When all else fails, take your equation to sines and cosines and then multiply through to clear your denominator(s), if any. Be sure to state any restrictions.

Writing your Solutions

The amount of time it takes to format your solution depends on whether the argument of the trigonometric function is θ or something other than θ.

	[0, 2π)	**All Solutions**
θ	List off all the solutions that lie in $[0, 2\pi)$.	List off all the solutions that lie in $[0, 2\pi)$, then after each one put "$+ 2\pi k$" where k is any integer.
Something other than θ	Take the argument and set it equal to each solution that lies in $[0, 2\pi)$, then after each one put "$+ 2\pi k$" where k is any integer. Next, solve for θ in the resulting equation(s). To generate only those solutions that lie in $[0, 2\pi)$, let $k = -1$, $k = 0$, $k = 1$, $k = 2$ and so on until you have some $\theta < 0$ (which reject), $0 \le \theta < 2\pi$ (accept these as your answers), and $\theta \ge 2\pi$ (which reject).	Take the argument and set it equal to each solution that lies in $[0, 2\pi)$ then after each one put "$+ 2\pi k$" where k is any integer. Lastly, solve for θ in the resulting equation(s).

Finally, apply your restrictions, and remember it's okay to have "No Solution" as an answer.